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A Design Procedure for a Tension-Wire Stiffened Truss-Column

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SUMMARY

A deployable, tension-wire stiffened truss-column configuration is considered for space structure applications. An analytical procedure is developed for design of the truss-column and exercised in numerical studies. The analytical procedure is based on using equivalent beam stiffness coefficients in the classical analysis for an initially imperfect beam-column. Failure constraints are then formulated which are used in a combined weight/strength and nonlinear mathematical programming automated design procedure to determine the minimum mass column for a particular combination of design load and length. Numerical studies show the mass characteristics of the truss-column for broad ranges of load and length. Comparisons of the truss-column with a baseline tubular column are made using a special structural efficiency parameter for this class of columns.

INTRODUCTION

One frequently considered concept for large space structures is a space truss composed of stiff column or bar elements. Design conditions for this type of structure have been considered in references 1 and 2. This concept is discussed in reference 3 for application to the solar array structure in a space solar power station.

Critical to this concept are the design and fabrication of the column or bar members making up the truss and efficient packaging for transporting the elements to space. References 1, 2, 3, and 4 have considered various aspects of this problem. Reference 5 presents a design procedure and structural efficiency comparison for several different column types to evaluate their suitability for space applications based on structural efficiency. However, the associated problems of fabrication and packaging of the columns are not considered in reference 5.

Because of the large size of many of the space systems being considered, both the length and diameter of the individual component members must be large to create an efficient structure. This large size for the column element makes complete fabrication to the structural state on Earth and subsequent transportation to space impractical. Thus fabrication and packaging of a column concept become at least as important as structural efficiency and probably more so.

Two basic fabrication/packaging concepts have received most of the attention in studies of large space systems. In one approach, the primary structural columns are assembled on orbit from Earth fabricated, nestable tapered columns (refs. 2 and 4). It has been shown in these references that, because of the nesting of the column half elements, high packing densities can be achieved. In addition, the tapered columns can be assembled into large primary columns that have good structural efficiency. The drawback of this method is that the on-orbit assembly is highly labor intensive requiring automation for the large systems. In the second approach, raw material stock (either aluminum or composites) is transported along with a fabrication machine to space, where the actual fabrication takes place (refs. 3 and 6). Although both structurally efficient members and high packing densities can be achieved with this approach, the added complexity of on-orbit fabrication is a significant drawback.

An alternative to these concepts is a deployable primary column which is fabricated, tested, and packaged on Earth and then transported into space. The column is then expanded to its structural state with a minimum of operations on orbit. This approach eliminates the time consuming assembly of many smaller components and minimizes the complicated operations in space associated with on-orbit fabrication. The problems of a deployable column concept are:

(1) The difficulty of designing a collapsable structure whose deployed length and possibly width are large compared with the launch vehicle.

(2) Maintaining high structural efficiency so that the overall mass that must be transported to space is low.

(3) Packaging the structure appropriately to achieve mass critical payloads in the launch vehicle.

In this study, attention is focused on problem (2). A tension-wire stiffened truss-column design, that was previously described in reference 7, is considered herein. Reference 7 presented results from a structural efficiency and packaging study of the column. Presented herein, are:

(a) A description of the load carrying mechanisms and structural behavior of the column.

(b) An analytical and design procedure for the column.

(c) A number of numerical studies which characterize the column design for a broad range of lengths and applied axial loadings.

SYMBOLS

A_ℓ, A_c, A_s, A_d	cross sectional areas of longerons, center column, spokes, and diagonals
a	maximum amplitude of sinusoidal imperfection
C	axial stiffness for the truss-column defined in equation A-1
C_s	transverse shear stiffness for the truss-column defined in equation A-4
D	bending stiffness for the truss-column defined in equation A-3
E_ℓ, E_c, E_s, E_d	Young's modulus of the longerons, center column, spokes, and diagonals
G_i	failure constraints on the design of the truss-column
I_c	moment of inertia of the center column
L	column length
M	column mass; also bending moment (equation A-5)

M_J	multiplier to account for the mass of the joints in the truss column
M_{max}	maximum bending moment induced in the column by an imperfection
n	number of bays in a truss-column
P	applied axial compressive load in the column
P_{des}	design load for the column (assumed compressive)
P_E	Euler buckling load for the column (equation A-6)
$P_{I\ell}$, P_{Ic}	initial forces in the longerons and center column
$\bar{P}_d$	axial component of the diagonal load at any column cross section
P_s	load induced in a spoke by the diagonals
R	radius of truss-column cross section (see figure 2)
r_ℓ, r_c, r_s, r_d	cross sectional radii of longerons, center column, spokes, and diagonals
r_{min}	minimum allowable tube radius for spokes and center column
$r_{sol,min}$	minimum allowable solid radius for members
T	initial tension in the diagonals
t_c, t_s	wall thickness of center column and spokes
V	transverse shear force induced by an imperfection
V_{max}	maximum value of the transverse shear force
x	column axial coordinate
Δ	axial end shortening of the column
η	ratio of longeron stiffness to center column stiffness defined in equation A-21

Π_c	total complementary potential energy of the column
θ	diagonal angle defined in figure 2
ε	column neutral axis strain
κ	column curvature
ρ_ℓ, ρ_d, ρ_c, ρ_s	material density of longerons, diagonals, center column and spokes

DESCRIPTION OF THE TENSION WIRE STIFFENED TRUSS-COLUMN

A model of a four bay section of a tension wire stiffened truss-column in its deployed state is shown in figure 1. A hinge joint or joints would be present along the center column in an actual structure to allow the packaged column to be folded to fit in the cargo area of the launch vehicle. In the analysis presented, a very simple butt hinge is postulated which will transmit compressive forces in the center column but not tensile forces. The requirement that the center column not be carrying a tensile load when the overall truss-column is under tension is prescribed as a design condition.

The tension-wire stiffened truss-column discussed herein is composed of four types of members:

(1) The center column runs along the centroidal axis of the equilateral triangular cross section and carries the compressive forces induced by applying initial tension to the longeron members plus a part of any axial loading applied to the truss-column.

(2) The three longeron members at the vertices of the triangular cross section are under an initial tensile load. These provide all of the truss column's bending stiffness and part of the axial stiffness until one or more of the three longerons become "slack" due to an applied compressive load to the column.

(3) The three sets of diagonal members in each bay of the truss-column are also capable of carrying only tension. These provide all of the transverse shear stiffness of the truss column until the shear load reaches a value that causes the diagonals to become "slack". Actually, under a shear load, some of the diagonals tend to lose tension and some become more stressed; however, a significant loss of transverse shear stiffness results when any of the diagonal members become slack.

(4) Three spokes are placed at intervals (the bay length) along the center column, and in the deployed state, run radially from the center column to the longerons. When the truss-column is compacted, the hinges at each end of the spoke allow the spoke to fold flat against the center column. At the same time, the longerons collapse onto the center column to drastically reduce the radius of the truss. The spokes serve to maintain the cross sectional shape, to support the center column at intervals, and also to induce a uniform tension in the diagonal members.

From figure 1 it can be seen that one of the interior spokes at each set is slightly buckled. This buckled spoke acts as a constant force spring which regulates the tension in the diagonal members. Thus, when the column is deployed and the spokes buckle, a uniform tension, related to the buckling load of the spoke, is induced in the diagonals. If the spokes remained rigid during deployment, the tension in the diagonals would be highly dependent on the exact length of the diagonal and due to manufacturing tolerances would vary from bay to bay. The design criteria for the internal spokes, then, is that their buckling load induce the necessary tension in the diagonals. The exterior spokes (three at each end of the column) can not buckle since they carry a vertical component of load from the longerons sloping to the center column. These three spokes at each end are designed specially.

ANALYSIS AND DESIGN PROCEDURE

The design of the wire-stiffened expandable truss-column is made more difficult because (1) certain considerations have to be made to allow the structure to deploy reliably and (2) since it is a pre-tensioned structure, the values of the initial forces must be selected and have an important influence on the remainder of the design. The approach used to analyze the truss-column is very similar to that used in reference 5 with the above two factors included. In reference 5 the importance of considering the effects of an imperfection in the column design was demonstrated. An imperfection or bow in the column has two primary implications. First, the imperfection causes a bending moment to be induced in the truss-column due to an axial load which tends to overload one of the longerons. Second, the imperfection causes a transverse shear force to be induced in the truss-column which must be carried by the diagonal members. This induced transverse shear force determines the amount of initial tension required in the diagonal members.

The analysis of the truss-column is based on determining a set of equivalent beam properties for the truss and using these in an imperfect column analysis (see ref. 8) to predict the overall loads and deformations. Then the forces in the individual members (center column, longerons, spokes, and diagonals) can be calculated and used to determine expressions for different failure modes. This analysis procedure is derived in detail in Appendix A.

Based on this analysis the design of the truss-column is accomplished by a combination of classical weight-strength methods and direct minimization of the truss mass to yield the minimum mass structure. The design load, P_{des}, the column length, L, the wall thickness of the tubular center column and spokes, and the diagonal angle, θ, (see fig. 2) were assumed to be known. Although the tubular member wall thicknesses could also be selected in the design process, experience has shown that for these lightly loaded columns, wall thicknesses tend to be selected at a minimum gage due to manufacturing constraints. Specific values taken for wall thicknesses are listed in the numerical results section. For manufacturing convenience, the diagonal angle, θ, was also kept constant in the optimization process; however, the

effect of variation in θ on the truss-column mass was considered in numerical studies. Because θ is constant, the number of bays, n, in the fixed length truss also fixes the cross sectional dimensions. The optimum number of bays, n, the longeron radius, r_ℓ, the center column radius, r_d, and the initial pretension in the column are the design variables considered. To maintain a realistic and manufacturable design, lower bounds on the member cross sectional properties were also assumed. For the spokes and center column, a constraint on minimum allowable tube radius is imposed. When the optimization routine tries to select a radius for these members less than this minimum, it is forced to consider the mass and stiffness of a solid member instead. Thus for a short, lightly loaded truss-column, all of the members could be solid rods. A minimum allowable radius is also prescribed for the solid members--longerons, diagonals, and spokes and center column for lightly loaded cases.

The mixed weight-strength and nonlinear mathematical programming approach to the design of the truss-column was chosen to exploit characteristics of this specific problem and to allow the flexibility to impose lower bounds or arbitrarily fix certain of the design variables. For example, the diagonals and spokes can be sized directly (see equations A-34, A-36, and A-37) based on assumed failure modes which eliminates their design from the mathematical programming process. Sizing of the longerons and center column is slightly more complicated, however, because of the interaction between these members. It was found that failure of these members was governed by two constraints: G_1, a constraint against the center column buckling between bays and G_2, a constraint requiring tension to be maintained in the longerons (see Appendix A for expressions for G_1 and G_2). However, for any value of n, these two constraints are not necessarily simultaneously active. The trends exhibited in the truss-column design process and the relationship between the constraints G_1 and G_2 can be seen in figure 3. For this particular case, the minimum mass truss-column has approximately 50 bays. The curve for the mass of the center column can be divided into two ranges; to the left of n = 50, the center column is buckling critical or constraint G_1 is active. In this range, the longeron "slackening" constraint G_2 is not active. To the right of n = 50, G_1 is no longer active and G_2 becomes the governing constraint.

It is obvious that for the optimum number of bays, n = 50, both constraints are active which is characteristic of a weight-strength design. However, in general, two other considerations are important:

(1) In practice, slightly off-optimum designs particularly with with fewer bays may be desirable.

(2) Minimum gage constraints becoming active for one or both of these members will influence constraints G_1 and G_2.

Thus the determination of the center column and longeron radii, subject to lower bound constraints, is best solved by a nonlinear math programming approach. Since the constraint equations are quite simple, they were directly coupled with an available computer code for constrained minimization, CONMIN (ref. 9).

STRUCTURAL BEHAVIOR OF THE TRUSS-COLUMN

Insight into the load carrying mechanism of the truss-column can be obtained by calculating the end deflections of the column under axial load. The load-shortening curve for an imperfect column can be determined from equation A-11. The corresponding loads in the center column, longerons, and diagonals under the applied axial load P can be found from equations A-26, A-28, A-14 and A-27, respectively. The load-deflection behavior for one specific case, a 50 m column with a 500 N design load, is shown in figure 4. The axial deflections of the overall column and members are plotted over the complete range of loading from $-P_{des}$ (tension) to P_{des} (compression). Because of the spokes acting as a regulating mechanism maintaining constant tension, the axial force in the diagonals, $\bar{P}_d$, is constant for all loadings. At $P = -P_{des}$ the force in the center column drops to zero and all the loading is being carried by the longerons. Similarly, when $P = P_{des}$, the most heavily loaded longeron becomes "slack" and will carry no additional load.

The nonlinear behavior of the truss-column due to the assumed imperfection is evident when P approaches P_{des}. However, the nonlinearity is fairly mild. The reason for this is that the optimum column designed for an imperfection tends

to have a value of P_{des}/P_E significantly less than 1.0. For this particular example P_{des}/P_E = .614. An efficient column design is achieved essentially by making P_{des}/P_E small enough so that nonlinear bending effects are small.

NUMERICAL STUDIES

Using the design procedure developed in Appendix A, numerical results were obtained for various values of the column length (5 m, 50 m, 500 m), design load (5 N, 50 N, 500 N, 5000 N, 25000 N), assumed imperfection a/L (.0001-.0060), and diagonal angle θ (15°-75°). A value of 110.2 GN/m^2 (16 x 10^6 psi) was used for the Young's modulus of the graphite/epoxy material in the center column, longerons, spokes and diagonals. This is typical of a material composed of mostly unidirectional, intermediate modulus fibers. The density is taken to be 1522 kg/m^3 (.055 lbm/in^3).

The selection of minimum gage dimensions for the truss members is somewhat arbitrary since it requires an assessment of the state of the art in manufacture of graphite/epoxy materials which is also highly dependent on both fiber and resin systems used. However, the following dimensions were selected for the numerical studies:

- tubular wall thickness, t_c, t_s = .71 mm (.028 in)
- minimum tube radius for spokes and center column, r_{min} = 2.54 mm (.1 in)
- minimum radius of solid cross section $r_{sol,min}$ = .381 mm (.015 in)

It was also necessary to select values for the column imperfection parameter a/L. In reference 8 where the parameter a/L as a measure of imperfections is discussed, the value a/L = .0025 is suggested for design calculations. However, this value was selected with reference to civil engineering applications and it is expected here that careful fabrication procedures which would be used for space applications would reduce the overall imperfections. Thus a value of a/L = .0010 was used for most of the cases.

As mentioned in Appendix A, the effect of joint mass was also considered by assuming the total mass of all joints to be a fixed percentage of the structural mass. Accounting for joint mass in this way has no effect on the

other design parameters but it does provide a means for making rational comparisons between the truss-column and other columns without joints. In equation A-38, the total mass of the truss-column M is defined to be a product of a joint mass factor M_J and the structural mass. A value of $M_J = 1.35$ was selected for the numerical studies.

Details of the designs for two values of diagonal angle (45°, 60°) are shown in Table I. The constraint imposed by assuming minimum gage dimensions is seen by noting that the diagonal radius r_d is usually designed by this value. The ratios of the design load to the Euler buckling load, P_{des}/P_E, for the more heavily loaded columns indicate the knockdown in load carrying capability due to imperfections. For the very lightly loaded columns, the longerons are often sized by the minimum gage value $r_{sol,min}$ which causes P_{des}/P_E to be overly low.

The effect of the value of a/L on the truss-column mass for one specific case is shown in figure 5. The mass penalty for a column designed for an a/L = .0010 compared with a/L = .0001 is approximately 18%. The number of bays in the optimum column is also shown for different designs. As can be seen, larger assumed imperfections tend to reduce the number of bays and thus increase the cross section and column bending stiffness.

Although a diagonal angle θ of 45° was selected as a baseline angle, it was recognized that this was not optimum for either structural efficiency or manufacturability. The effect of diagonal angle θ on the truss-column mass is shown in figure 6. More structurally efficient columns have values of θ less than 45° and necessarily more bays. However, it is expected that the cost of manufacturing is directly proportional to the number of bays (and joints) and thus a higher value of θ might be more desirable. Comparisons of the number of bays for designs with θ = 60° versus θ = 45° for different column lengths and loads can be made from Table I.

Finally, to compare the tension wire stiffened truss-column with other column concepts, a structural efficiency plot for the truss-column and tubular column is presented in figure 7. The structural efficiency parameter $M/L^{5/3}$ was shown in reference 5 to be useful for comparing the masses of lightly loaded columns at different values of the design load. A tubular column

design appears as a single, length independent line on a structural efficiency plot using this parameter. This allows for easy comparison between the tubular column and other column concepts such as the wire stiffened truss-column. The tubular column is of interest not only as a structural baseline but also because its structural efficiency is very close to that of the double tapered, nesting graphite/epoxy column described in references 2 and 4. The graphite/epoxy tubular column plot in figure 7 represents an optimum design which is critical in Euler buckling and has a wall thickness t = .015 and modulus equal to that of the truss-column. For higher loadings, local wall buckling would have to be considered; the point where this would be necessary and the structural efficiency of columns in this range are indicated approximately by the dashed line in figure 7.

Structural efficiency curves for truss-columns of three lengths (5 m, 50 m, 500 m) are also shown in figure 7. A diagonal angle θ of 45° and an assumed imperfection a/L of .0010 were prescribed. The flattening out of the curves for the 5 and 50 meter columns at low loadings is because the minimum gage parameters are becoming active for these designs. Over most of the load range, the 5 m truss-column is more efficient than the tubular column; for the longer lengths (50 m, 500 m) this is even more significant. Structural efficiency plots of this type for other column concepts can be found in reference 5 which can be used in comparisons with the tension stiffened truss-column in this study.

CONCLUDING REMARKS

The structural efficiency of a wire-stiffened, expandable truss-column is investigated by developing a design procedure for a minimum mass structure. The design procedure employs a combined weight-strength and direct mass minimization approach exploiting specific characteristics of the structure. Numerical results are obtained for truss-columns with a wide variety of lengths and design loads and the masses are compared with tubular column designs. Specific conclusions obtained from this study are as follows:

(1) The wire-stiffened, expandable truss-column is significantly more structurally efficient than the baseline tubular column, particularly for longer lengths.

(2) Design based on an assumed imperfection is important because (a) it provides a criteria for determining the necessary value for the diagonal number pretension, and (b) significant differences in both the final configuration and mass of the structure compared with the perfect case result.

(3) For short or lightly loaded columns the design may be dictated by manufacturing minimum gage constraints and these must be included.

(4) For realistic automated design, the design/analysis procedure should be sufficiently general to allow arbitrary bounds on design variables and a general set of constraint functions. Nonlinear mathematical programming methods allow the criteria to be met.

APPENDIX A

ANALYSIS OF THE TRUSS COLUMN

The analysis of the tension wire stiffened truss-column shown in figures 1 and 2 is based on determining a set of equivalent beam extensional, bending, and transverse shear stiffness for the truss. These equivalent stiffnesses are used in the imperfect column analysis to predict the overall behavior of the structure. The individual member loads in the center column, longerons, diagonals, and spokes are then calculated from the overall loads and deformations of the column. Knowing the member loads, the failure of each component can be predicted. Finally, the optimum truss-column is designed by minimizing the truss mass subject to constraints on the different failure modes and side constraints on the design variables.

Equivalent Beam Stiffnesses for the Truss

The axial and bending stiffness for the truss-column are derived from simple statics. Because the spokes are buckled in the unloaded column, the diagonal members and the spokes do not contribute to the axial or bending stiffnesses. The equivalent axial stiffness, C, can be written as

$$C = 3E_{\ell}A_{\ell} + E_{c}A_{c} \tag{A-1}$$

Noting that the center column lies at the centroid of the triangles which is the neutral axis for the beam and neglecting the bending stiffnesses of the individual members, the bending stiffness, D, can be written as

$$D = \frac{3E_{\ell}A_{\ell}R^2}{2} \tag{A-2}$$

where R is shown in figure 2.

This can be rewritten in terms of the overall column length, L, the number of bays in the column, n, and the diagonal angle, θ, as

$$D = \frac{E_{\ell}A_{\ell}L^2}{2n^2\tan^2\theta} \tag{A-3}$$

The transverse shear stiffness, C_s, is obtained from reference 10 as

$$C_s = 3A_d E_d \sin\theta \cos^2\theta \qquad \text{(A-4)}$$

The transverse shear stiffness will be specified later so that the transverse shear deflections in the column are negligible compared with those due to bending. In the column analysis that follows, the effect of transverse shear deformation is ignored.

Column Analysis with an Initial Imperfection

In reference 5, the effects of an initial imperfection in column straightness were considered in the design of truss-columns. In the present study a similar approach is used for the tension-wire stiffened column. The two primary implications of such an imperfection are that a bending moment and a transverse shear force are induced in the column in addition to the axial force. The moment induced in an axially loaded column with a sinusoidal imperfection in straightness is taken from reference 8 as

$$M = \frac{Pa}{1 - P/P_E} \sin\frac{\pi x}{L} \qquad \text{(A-5)}$$

where P is the axial load, a is the maximum amplitude of the initial imperfection, x is an axial coordinate referenced from the end of the column, and P_E is the Euler buckling load of a perfect column which is defined as

$$P_E = \frac{\pi^2 D}{L^2} \qquad \text{(A-6)}$$

The maximum moment occurs at the center of the column ($x = L/2$) and is obtained from equation A-5 as

$$M_{max} = \frac{Pa}{1 - P/P_E} \qquad \text{(A-7)}$$

The imperfection induced shear load V is determined by taking the derivative of equation A-5 with respect to x. This results in

$$V = \frac{P \frac{\pi a}{L}}{1 - P/P_E} \cos \frac{\pi x}{L} \tag{A-8}$$

which has a maximum value at (x = 0) of

$$V_{max} = \frac{P \frac{\pi a}{L}}{1 - P/P_E} \tag{A-9}$$

For the imperfect column, the end shortening is a function of both the strain along the column neutral axis and the transverse deflection, w. This end shortening (Δ) under the axial load is a measure of the column's stiffness which is of significance when considering a truss structure assembled from these members. An expression for the end shortening, consistent with equation A-5, can be derived by considering the nonlinear strain-displacement relation for a column with a sinusoidal imperfection

$$\varepsilon = \frac{du}{dx} + \frac{1}{2}\left(\frac{dw}{dx}\right)^2 + \frac{a\pi}{L}\frac{dw}{dx}\cos\frac{\pi x}{L} \tag{A-10}$$

where $\varepsilon = \frac{-P}{C}$ and since $M = -D\frac{d^2w}{dx^2}$, $w = \frac{Pa}{P_E - P}\sin\frac{\pi x}{L}$

Defining the end shortening $\Delta = u(o) - u(L) = -\int_o^L \frac{du}{dx}\,dx$,

equation A-10 can be rearranged and integrated to give

$$\Delta = \frac{PL}{C} + \left(\frac{a}{L}\right)^2 \frac{\pi^2 L}{2} \frac{P/P_E\,(1 - P/2P_E)}{(1 - P/P_E)^2} \tag{A-11}$$

Member Loads in the Truss

Now that the stress resultants have been determined from the beam-column analysis, the loads in the longerons, the center column, the diagonals, and spokes can be calculated. As mentioned previously, the diagonals and spokes do not carry any of the bending or axial loading in the column; the forces in these members are due only to the induced transverse shear and initial

forces. Because the center column lies along the beam's neutral axis it carries none of the induced bending moment. Its load is due only to the column axial force. The forces in the longerons are due to the column axial force and the imperfection induced bending moment. The derivation of expressions for these member forces is considered below.

Since the internal equilibrium of the truss-column is a statically indeterminate problem, the member forces are determined by considering the deformations and axial equilibrium at any point along the length. The force in any longeron can be written in terms of the neutral axis strain, ε, and curvature, κ, as

$$P_{\ell} = A_{\ell}E_{\ell}\varepsilon + A_{\ell}E_{\ell}R\kappa + P_{I\ell} \tag{A-12}$$

where $P_{I\ell}$ is the as yet undetermined initial force in the longeron. The force in the center column can be written similarly as

$$P_c = A_cE_c\varepsilon + P_{Ic} \tag{A-13}$$

where P_{Ic} is the initial force in the column. The diagonal members also have an axial force component which must be considered for equilibrium. If the diagonal members are assumed to be carrying a tensile force, T, the total axial force component due to all six diagonals at any column cross section is

$$\bar{P}_d = -6T \sin\theta \tag{A-14}$$

Because the spokes are initially buckled, this axial component, $\bar{P}_d$, remains constant for all loadings.

For equilibrium of the unloaded column,

$$\bar{P}_d + 3P_{\ell} + P_c = 0 \tag{A-15}$$

when $\varepsilon = \kappa = 0$.

Substituting equations A-12, A-13, and A-14 into A-15 gives

$$P_{I\ell} = \frac{6T \sin\theta - P_{Ic}}{3} \tag{A-16}$$

From the beam-column analysis the equilibrium equations can be written as

$$\varepsilon = \frac{P}{C} \tag{A-17}$$

$$\kappa = \frac{M_{max}}{D} \tag{A-18}$$

where P is the total axial load on the column and M_{max} is the moment due to the imperfection and is defined in equation A-7. Substituting equations A-16, A-17, A-18, and A-7 into equations A-12 and A-13 gives the force in the most highly loaded longeron as

$$P = \underbrace{\frac{P}{3(\eta+1)}}_{\text{due to axial force}} + \underbrace{\frac{2}{3R}\frac{Pa}{(1 - P/P_E)}}_{\text{due to bending moment}} + \underbrace{2T \sin\theta - \frac{P_{Ic}}{3}}_{\text{initial force}} \tag{A-19}$$

and the force in the column as

$$P_c = \underbrace{\frac{P}{\eta+1}}_{\text{due to axial force}} + \underbrace{P_{Ic}}_{\text{initial force}} \tag{A-20}$$

where η is a nondimensional stiffness parameter defined as

$$\eta = \frac{3A_\ell E_\ell}{A_c E_c} \tag{A-21}$$

The force in a single diagonal can be calculated by considering transverse equilibrium of any beam cross section as

$$P_d = \frac{V_{max}}{4 \cos\theta \cos 30^\circ} - T \tag{A-22}$$

which after substitution of equation A-9 becomes

$$P_d = \frac{P\left(\frac{a}{L}\right)}{2\sqrt{3} \cos\theta \, (1 - P/P_E)} - T \tag{A-23}$$

The force in the spokes can be calculated by considering equilibrium between the two sets of diagonals and the spoke and is given by

$$P_s = 2\sqrt{3}\ T \cos \qquad \text{(A-24)}$$

The forces in all four components of the truss-column (longerons, center column, diagonals, and spokes) have been related to the applied axial force, P, and the initial member forces, P_{Ic} and T. The values of these initial forces are selected by considering specific design conditions for the truss-column.

Design Conditions and Constraint Equations

To determine the initial forces T and P_{Ic} and to design the minimum mass truss-column for given values of P and column length, L, the following design conditions are imposed:

(1) When the beam is loaded with an applied tensile force equal to $-P_{des}$, the force in the center column P_c is zero. This insures that any hinge joints along the center column will not be put in tension.

(2) When the applied compressive load P is equal to P_{des}, the center column should not buckle between the spoke supports.

(3) When $P = P_{des}$, the longerons must not be in compression.

(4) When $P = P_{des}$, the induced transverse shear force at the beam ends causes the force in one set of diagonals to drop to zero.

(5) At $P = P_{des}$, 2% transverse shear deformation is allowed. The diagonals are sized to provide the necessary shear stiffness for this criteria to be met.

(6) The spokes are designed to buckle under the load induced by the two sets of diagonal members connected at each spoke.

Design condition 1 is applied to equation A-20 to find the initial column force, P_{Ic}, by setting $P = -P_{des}$

$$P_{Ic} = \frac{P_{des}}{\eta+1} \tag{A-25}$$

Equation A-20 now becomes

$$P_c = \frac{P}{\eta+1} + \frac{P_{des}}{\eta+1} \tag{A-26}$$

From condition 4 the initial tension T can be found by setting $P = P_{des}$ and $P_d = 0$ in equation A-23. The result is

$$T = \frac{P_{des}\left(\frac{a}{L}\right)}{2\sqrt{3}\cos\theta\,(1 - P_{des}/P_E)} \tag{A-27}$$

Substituting equations A-25 and A-27 into equation A-19 and replacing $R = L/(n\sqrt{3}\tan\theta)$ yields the force in the longeron

$$P_L = \frac{P - P_{des}}{3(\eta+1)} + \frac{\left(\frac{a}{L}\right)\tan\theta}{\sqrt{3}} \quad \frac{2nP}{(1 - P/P_E)} \quad \frac{P_{des}}{(1 - P_{des}/P_E)} \tag{A-28}$$

Given the member forces, P_d, P_s, P_c, P_L, in terms of a design load, P_{des}, an applied load, P, and the geometric properties of components, the minimum mass truss can be designed. The objective is to minimize the mass of the structure subject to both equality and inequality constraints. A common nomenclature for inequality constraints is used where

$G_j(v) \leq 0$; constraint satisfied

$G_j(v) > 0$; constraint violated

The equality and inequality constraints arise from the satisfaction of the six design conditions. Since condition 1 has already been used to find the initial column force, P_{Ic}, condition 2 is considered next.

The buckling load of the center column between spoke supports can be written as

$$P_{crit,\text{center column}} = \frac{\pi^2 E_c I_c n^2}{L^2} \tag{A-29}$$

where E_c is the modulus and I_c is the moment of inertia of the center column.

Condition 2 can be expressed by setting $P = P_{des}$ in equation A-27 and using equation A-29 as

$$G_1 = \frac{\frac{2P_{des}}{1+\eta}}{\pi^2 E_c I_c n^2/L^2} - 1.0 \tag{A-30}$$

In calculating G_1, the center column moment of inertia I_c is calculated for either an expression for a hollow thin walled tube or a solid circular member depending on the particular design conditions.

Condition 3 can be written similarly by setting $P = P_{des}$ in equation A-28 as

$$G_2 = \frac{\frac{P_{des}(1-\eta)}{3(1+\eta)}}{\frac{P_{des}\left(\frac{a}{L}\right)\tan\theta\ (\pi+2n)}{\sqrt{3}\ (1 - P/P_E)}} - 1.0 \tag{A-31}$$

Condition 4 was satisfied by selection of the initial tension force, T, in the diagonal members. These diagonal members still remain to be sized, however, which will be done by considering condition 5.

To insure that transverse shear deformation has a small effect on the buckling of the truss-column, the diagonal member area is selected so that the classical Euler buckling load is reduced only by 2%. From reference 8, the buckling load for a shear flexible column is

$$P_{cr} = \frac{P_E}{1 + \frac{P_E}{C_s}} \tag{A-32}$$

where P_E is given by equation A-6 and C_s, the transverse shear stiffness is given by equation A-4. The 2% reduction requires that

$$\frac{P_E}{C_s} = .02 \tag{A-33}$$

Substituting equation A-4 into A-33 and rearranging yields an expression for the diagonal member area

$$A_d = \frac{50}{3} \frac{P_E}{E_d \sin\theta \cos^2\theta} \tag{A-34}$$

In dealing with the imperfect column this equation is modified slightly to yield the actual expression used for determining diagonal member area

$$A_d = \frac{50}{3} \frac{P_{des}}{E_d \sin\theta \cos^2\theta} \tag{A-35}$$

Finally, the spokes are designed by considering condition 6. This is done by equating the induced load in the spoke, equation A-24, to an equation for the Euler buckling load of a simply supported tubular or solid circular member. Just as in the case of the center column, the choice of tubular or solid member depends on the specific design conditions--design load, column length, allowable wall thickness, etc. The two sets of equations governing the spoke design are:

Tubular member:

$$r_s = \left(\frac{P_s L^2}{3\pi^3 E_s t_s n^2 \tan^2\theta}\right)^{1/3} \tag{A-36}$$

$$A_s = 2\pi r_s t_s$$

Solid member:

$$r_s = \left(\frac{4 P_s L^2}{3\pi^3 E_s n^2 \tan^2\theta}\right)^{1/4} \qquad \text{(A-37)}$$

$$A_s = \pi r_s^2$$

By applying conditions 1-6, either direct expressions or constraint equations have been written governing design of components of the truss column. It should be noted that by the use of equations A-35 and A-36 or A-37 the diagonals and spokes can be sized directly. However, two design variables, the longeron radius r_ℓ and the center column radius r_c, remain undetermined and must be selected to give an overall minimum mass. The overall design problem, then, is a combination of both weight-strength design methods and direct minimization of the structure mass. The undetermined variables r_c and r_ℓ are determined by minimizing the truss mass which can be written

$$M = M_J L(3\rho_\ell A_\ell + \rho_c A_c + 6\rho_d A_d/\sin\theta + \sqrt{3}\,\rho_s A_s/\tan\theta) \qquad \text{(A-38)}$$

where A_d and A_s are found from equations A-35 and A-36 or A-37. The factor M_J is used to account for the mass of the joints in the truss and can be selected arbitrarily since it has no effect on the design and is used only to produce a realistic total mass for the truss.

Equation A-38 can be minimized to determine r_c and r_ℓ subject to constraints G_1 and G_2 (equations A-30 and A-31) and lower bounds on r_c and r_ℓ using nonlinear mathematical programming techniques. In this study an available computer code, CONMIN (ref. 9), based on a feasible direction method was used to perform the numerical studies.

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TABLE I. - DETAILS OF MINIMUM MASS GRAPHITE, TENSION-WIRE STIFFENED TRUSS-COLUMNS

P_{des} (N)	M (kg)	n	R (m)	r_t (mm)	r_c (mm)	r_d (mm)	r_s (mm)	η	P_{des}/P_E
L = 5m				a/L = .0010 θ = 45°					
5.	.079	25	.115	.381	.725	.381	.381	.829	.013
50.	091	49	.059	.381	.953	.381	.381	.484	.484
500.	.195	23	.125	.476	2.54	.381	.549	.189	.682
5000.	.658	19	.152	1.19	5.72	.825	1.13	.525	.743
25000.	2.00	12	.241	1.53	13.8	1.85	2.66	.358	.901
L = 50m									
5.	1.17	89	.324	.381	1.31	.381	.381	.254	.160
50.	1.96	72	.401	.497	2.55	.381	.526	.205	.614
500.	5.34	50	.577	1.11	6.66	.381	1.11	.394	.589
5000.	16.8	31	.931	1.97	19.6	.825	2.74	.417	.726
25000.	42.3	22	1.31	3.01	41.8	1.85	6.32	.459	.780
L = 500m									
5.	23.5	151	1.91	.467	3.41	.381	.558	.134	.306
50.	61.3	110	2.62	.917	8.92	.381	1.22	.199	.422
500.	167.	75	3.85	1.82	24.0	.381	2.67	.260	.549
5000.	506.	49	5.89	3.29	68.5	.825	8.62	.334	.650
25000.	1157.	36	8.02	5.26	141.	1.85	18.8	.414	.686
L = 5m				a/L = .0010 θ = 60°					
5.	.070	22	.076	.381	.768	.381	.381	.738	.029
50.	.099	32	.052	.382	1.22	.381	.381	.294	.617
500.	.208	19	.087	.644	2.79	.381	.495	.314	.763
5000.	.809	12	.138	1.26	7.96	1.05	1.16	.419	.801
25000.	2.53	7	.238	1.53	20.4	2.36	2.79	.244	.915
L = 50m									
5.	1.20	73	.228	.383	1.46	.381	.381	.206	.320
50.	2.29	47	.355	.601	3.36	.381	.473	.227	.538
500.	6.30	31	.538	1.15	9.39	.381	1.10	.297	.639
5000.	20.6	19	.877	2.04	27.9	1.05	2.76	.314	.763
25000.	52.0	14	1.19	3.33	57.1	2.36	5.90	.410	.777
L = 500m									
5.	28.5	92	1.81	.496	4.79	.381	.542	.109	.302
50.	74.0	68	2.45	.987	12.4	.381	1.17	.166	.417
500.	204.	47	3.55	1.90	33.6	.381	2.60	.227	.538
5000.	620.	31	5.38	3.63	93.9	1.05	8.04	.297	.639
25000.	1415.	22	7.58	5.37	201.	2.36	19.2	.303	.736

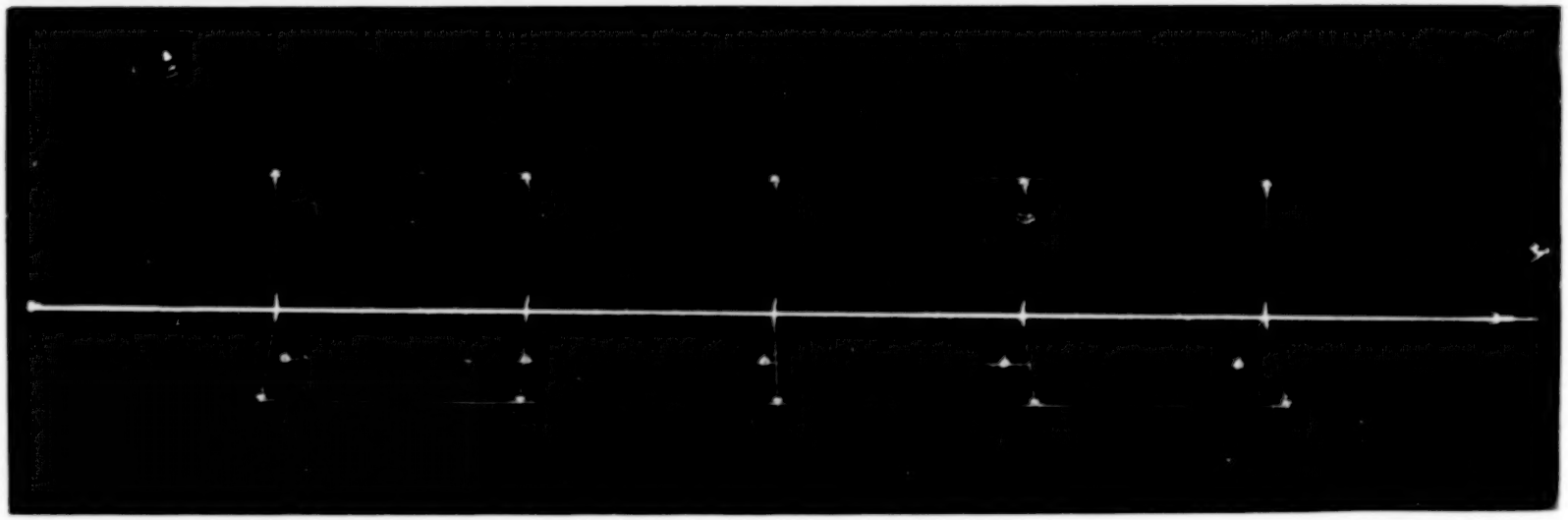

Figure 1. - Model of wire-stiffened expandable truss-column.

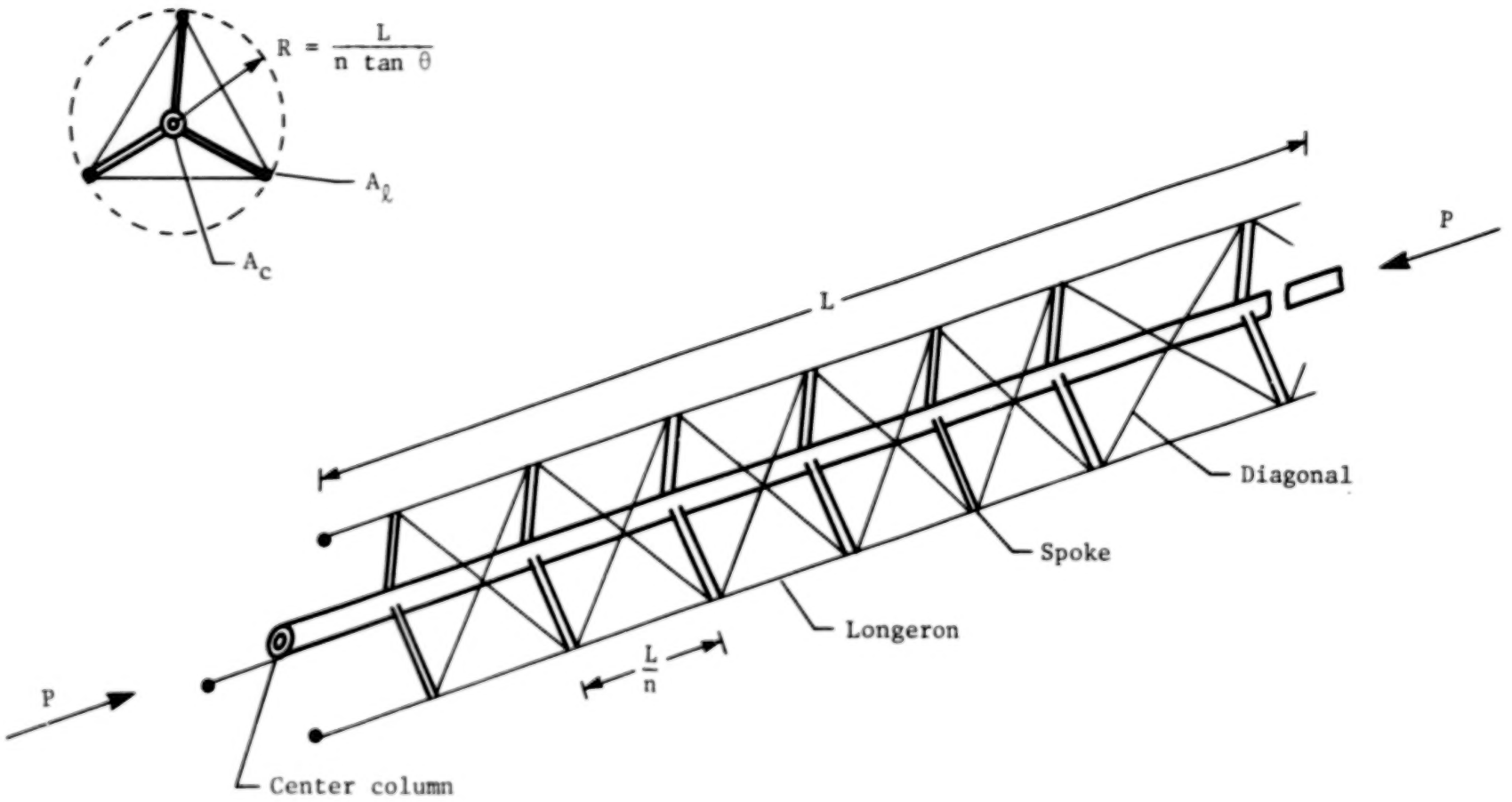

Figure 2.- Geometry of the tension-wire stiffened column.

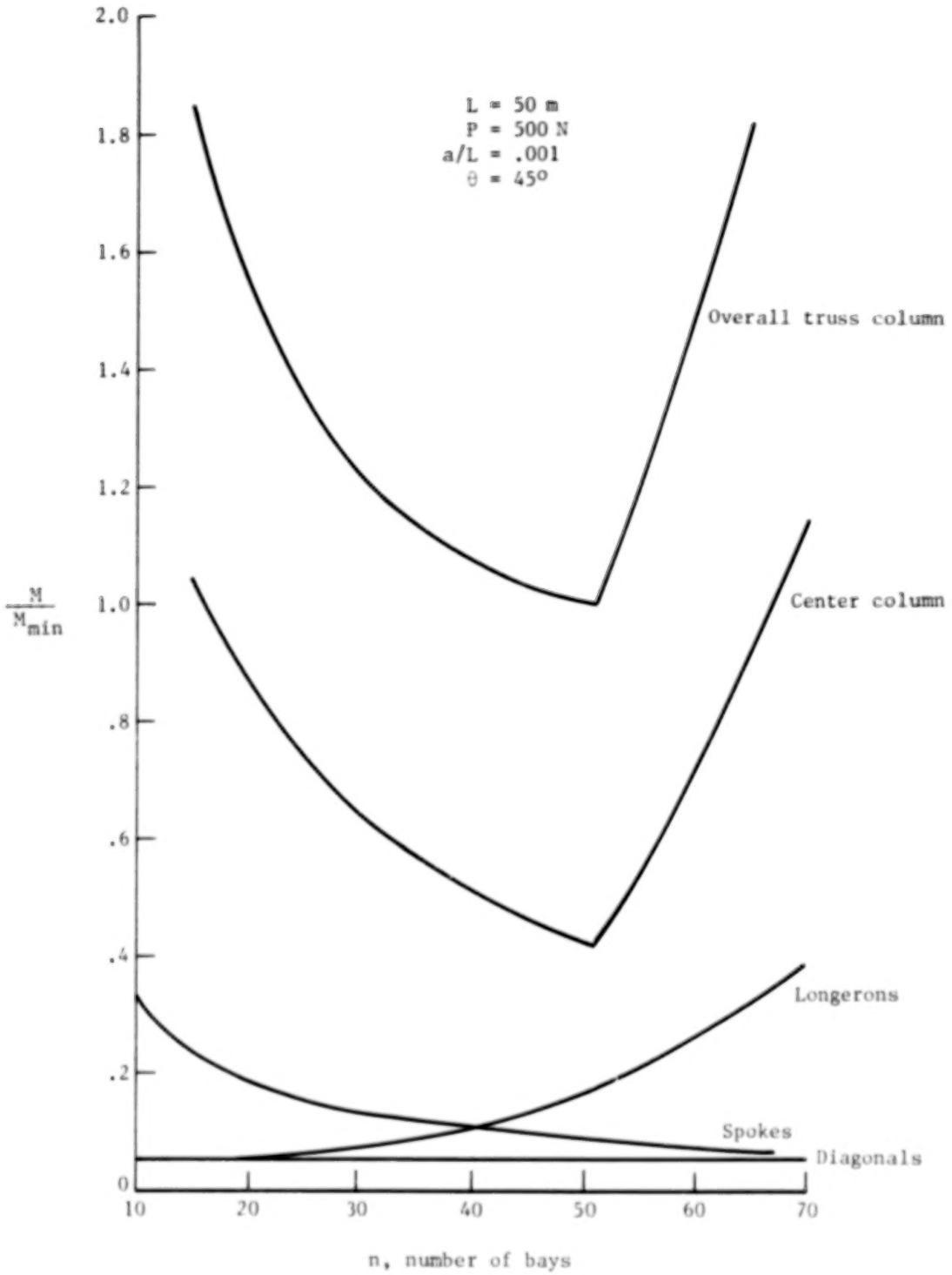

Figure 3.- Optimum mass distributions for the truss-column components.

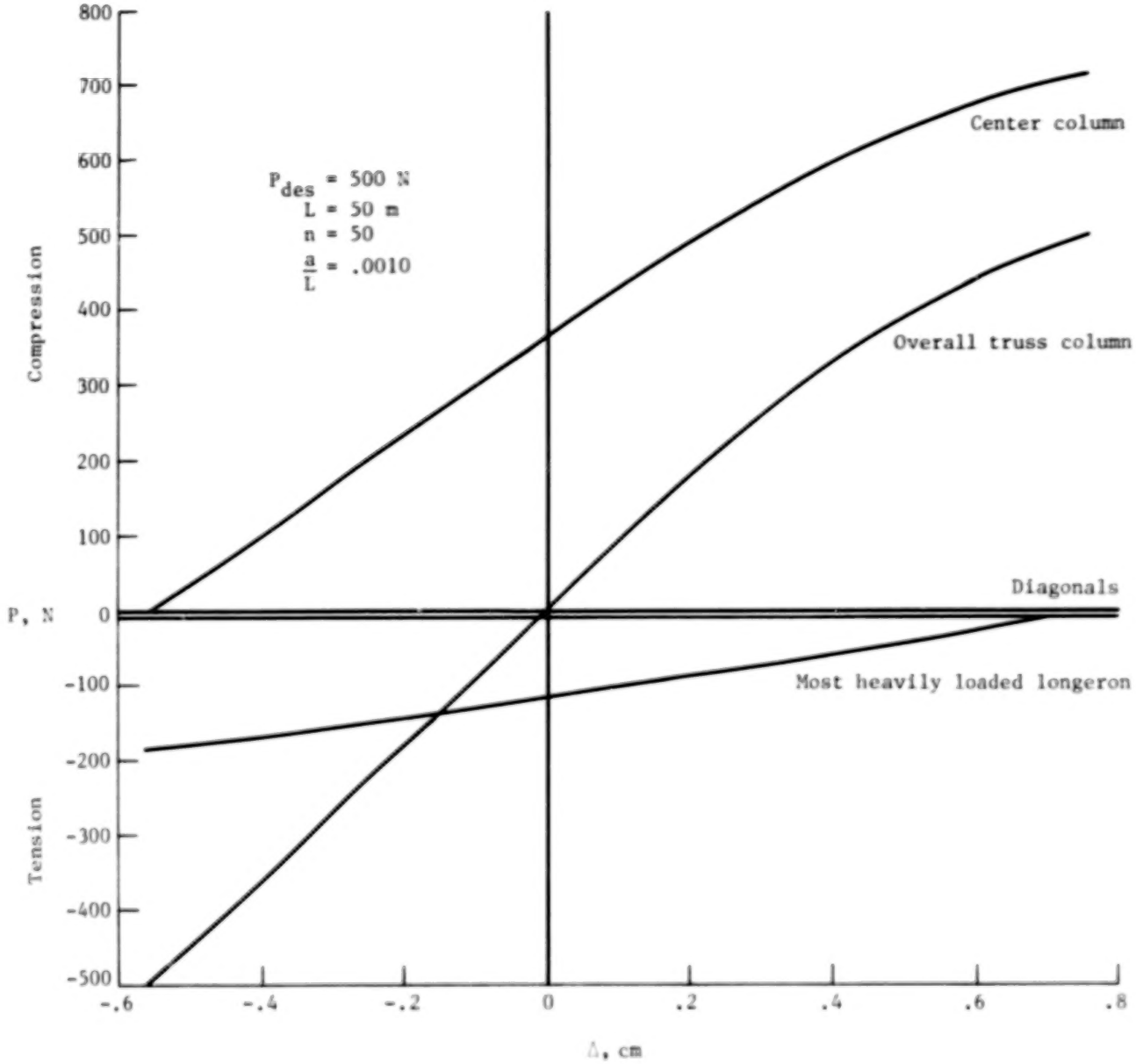

Figure 4.- Load-deflection paths for truss column and components.

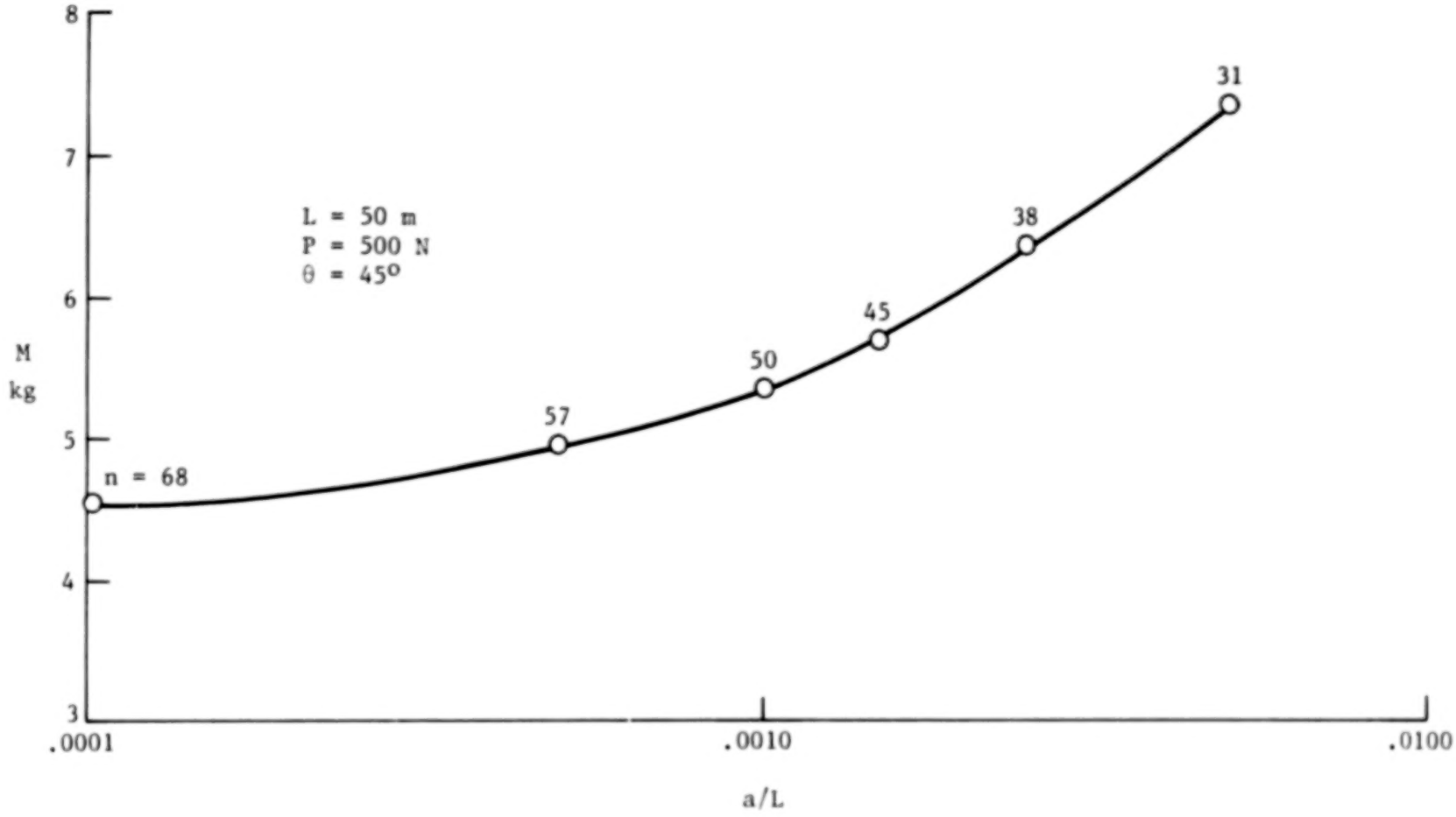

Figure 5.- Change in mass of the truss column as a function of initial imperfection.

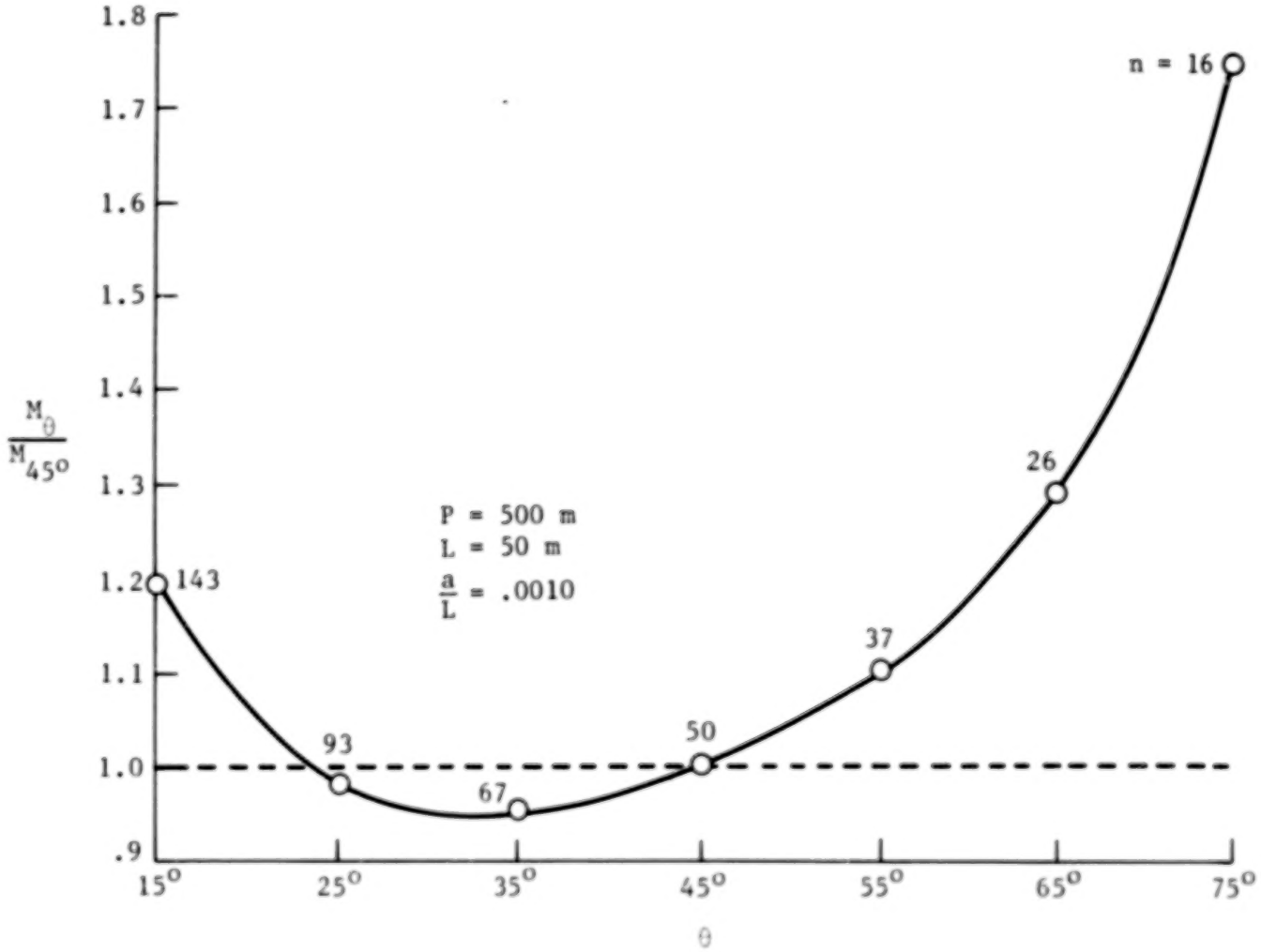

Figure 6.- Effect of diagonal angle on column mass.

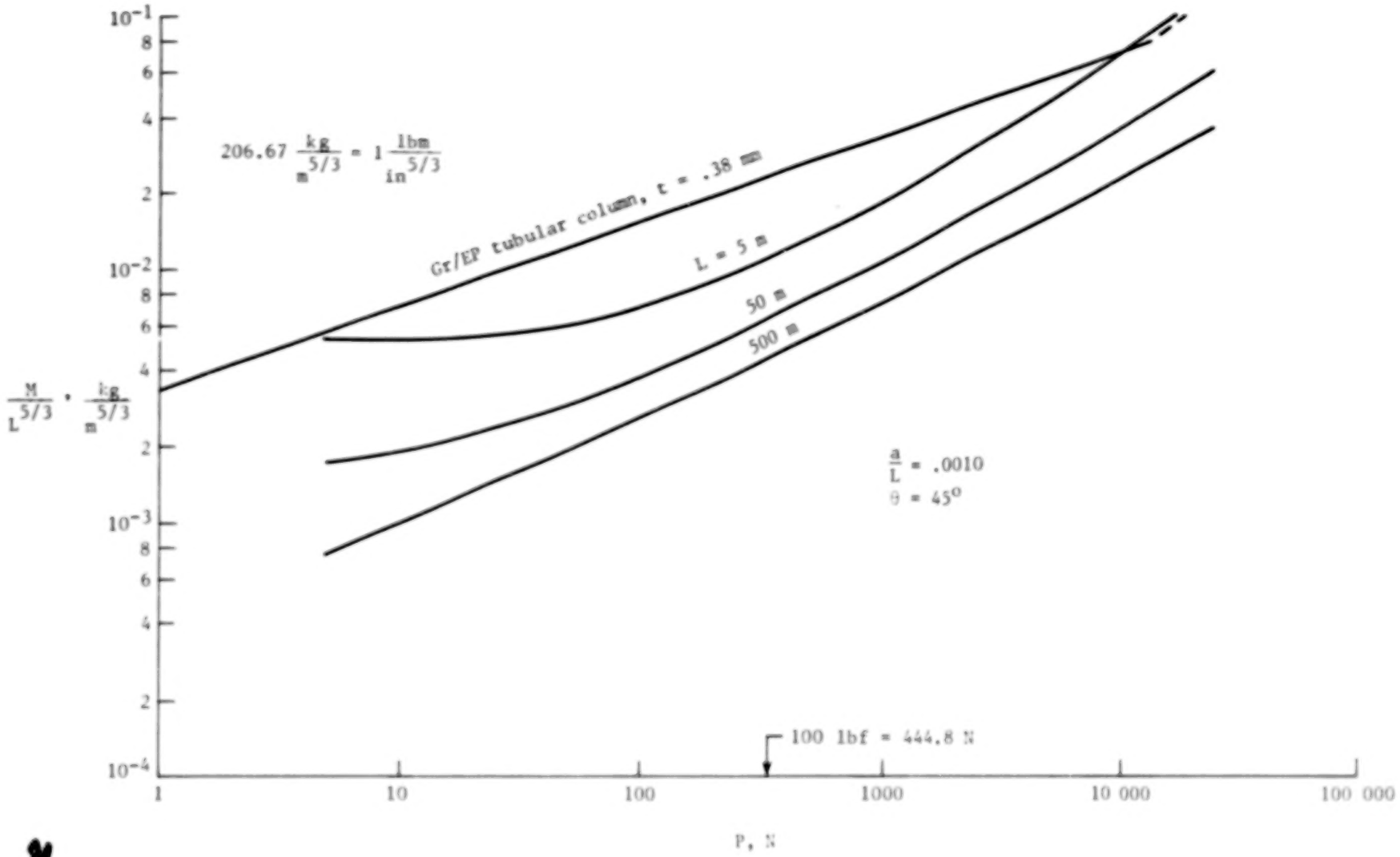

Figure 7.- Structural efficiency characteristics of the truss-column compared with tubular column.

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16. Abstract

A deployable, tension-wire stiffened truss-column configuration is considered for space structure applications. An analytical procedure is developed for design of the truss-column and exercised in numerical studies. The analytical procedure is based on using equivalent beam stiffness coefficients in the classical analysis for an initially imperfect beam-column. Failure constraints are then formulated which are used in a combined weight/strength and nonlinear mathematical programming automated design procedure to determine the minimum mass column for a particular combination of design load and length. Numerical studies show the mass characteristics of the truss-column for broad ranges of load and length. Comparisons of the truss-column with a baseline tubular column are made using a special structural efficiency parameter for this class of columns.

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